Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 2

Wednesday 11 May 2016 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 14]

180 people were interviewed and asked what types of transport they had used in the last year from a choice of airplane (A), train (T) or bus (B). The following information was obtained.

47 had travelled by airplane
68 had travelled by train
122 had travelled by bus
25 had travelled by airplane and train
32 had travelled by airplane and bus
35 had travelled by train and bus
20 had travelled by all three types of transport

(a) Draw a Venn diagram to show the above information. [4]

(b) Find the number of people who, in the last year, had travelled by

(i) bus only;

(ii) both airplane and bus but not by train;

(iii) at least two types of transport;

(iv) none of the three types of transport. [6]

A person is selected at random from those who were interviewed.

(c) Find the probability that the person had used only one type of transport in the last year. [2]

(d) Given that the person had used only one type of transport in the last year, find the probability that the person had travelled by airplane. [2]

2. [Maximum mark: 15]

Prachi is on vacation in the United States. She is visiting the Grand Canyon.

When she reaches the top, she drops a coin down a cliff. The coin falls down a distance of 5 metres during the first second, 15 metres during the next second, 25 metres during the third second and continues in this way. The distances that the coin falls during each second forms an arithmetic sequence.

(a) (i) Write down the common difference, d, of this arithmetic sequence.

(ii) Write down the distance the coin falls during the fourth second. [2]

(b) Calculate the distance the coin falls during the 15th second. [2]

(c) Calculate the **total** distance the coin falls in the first 15 seconds. Give your answer in kilometres. [3]

Prachi drops the coin from a height of 1800 metres above the ground.

(d) Calculate the time, to the nearest second, the coin will take to reach the ground. [3]

Prachi visits a tourist centre nearby. It opened at the start of 2015 and in the first year there were $17\,000$ visitors. The number of people who visit the tourist centre is expected to increase by $10\,\%$ each year.

(e) Calculate the number of people expected to visit the tourist centre in 2016. [2]

(f) Calculate the total number of people expected to visit the tourist centre during the first 10 years since it opened. [3]

3. [Maximum mark: 11]

A speed camera on Peterson Road records the speed of each passing vehicle. The speeds are found to be normally distributed with a mean of $67\,\text{km h}^{-1}$ and a standard deviation of $3.4\,\text{km h}^{-1}$.

(a) Sketch a diagram of this normal distribution and shade the region representing the probability that the speed of a vehicle is between 60 and $70\,\text{km h}^{-1}$. [2]

A vehicle on Peterson Road is chosen at random.

(b) Find the probability that the speed of this vehicle is

(i) more than $60\,\text{km h}^{-1}$;

(ii) less than $70\,\text{km h}^{-1}$;

(iii) between 60 and $70\,\text{km h}^{-1}$. [3]

It is found that $19\,\%$ of the vehicles are exceeding the speed limit of $s\ \text{km h}^{-1}$.

(c) Find the value of s, correct to the nearest integer. [2]

There is a fine of $\text{US\$65}$ for exceeding the speed limit on Peterson Road. On a particular day the total value of fines issued was $\text{US\$14\,820}$.

(d) (i) Calculate the number of fines that were issued on this day.

(ii) Estimate the total number of vehicles that passed the speed camera on Peterson Road on this day. [4]

4. [Maximum mark: 16]

A playground, when viewed from above, is shaped like a quadrilateral, $ABCD$, where $AB = 21.8\,\text{m}$ and $CD = 11\,\text{m}$. Three of the internal angles have been measured and angle $ABC = 47^\circ$, angle $ACB = 63^\circ$ and angle $CAD = 30^\circ$. This information is represented in the following diagram.

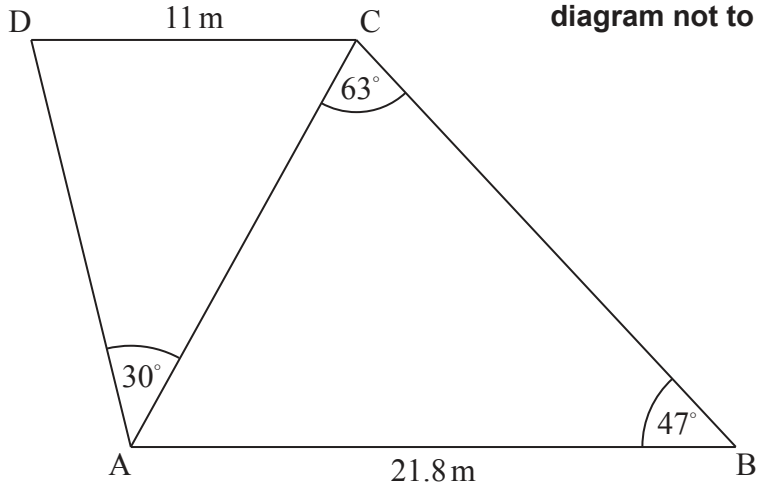


diagram not to scale

(a) Calculate the distance AC. [3]

(b) Calculate angle ADC. [3]

There is a tree at C, perpendicular to the ground. The angle of elevation to the top of the tree from D is 35°.

(c) Calculate the height of the tree. [2]

Chavi estimates that the height of the tree is $6\,\text{m}$.

(d) Calculate the percentage error in Chavi's estimate. [2]

Chavi is celebrating her birthday with her friends on the playground. Her mother brings a 2 litre bottle of orange juice to share among them. She also brings **cone-shaped** paper cups.

Each cup has a vertical height of $10\,\text{cm}$ and the top of the cup has a diameter of $6\,\text{cm}$.

(e) Calculate the volume of one paper cup. [3]

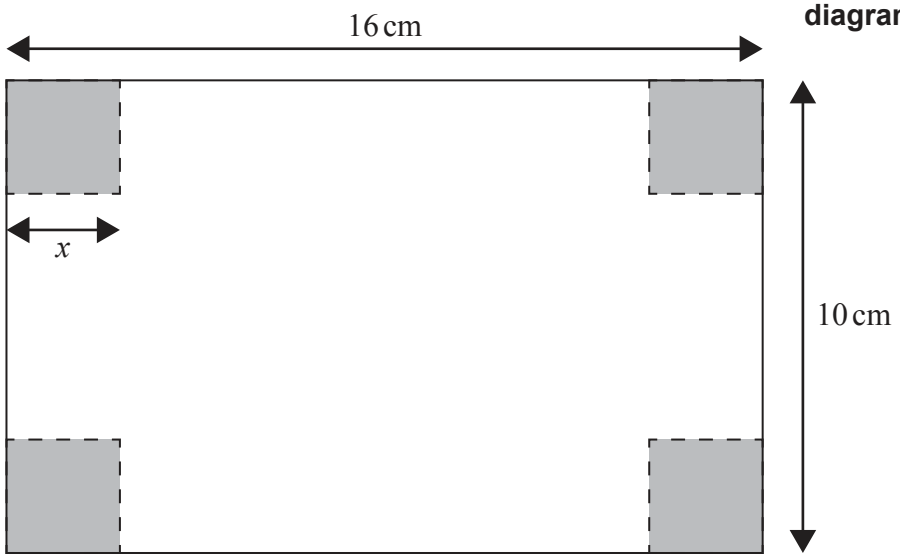
(f) Calculate the maximum number of cups that can be completely filled with the 2 litre bottle of orange juice. [3]

5. [Maximum mark: 19]

Hugo is given a rectangular piece of thin cardboard, $16\,\text{cm}$ by $10\,\text{cm}$. He decides to design a tray with it.

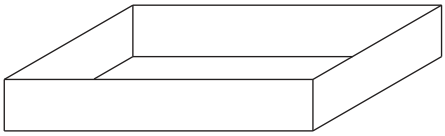
He removes from each corner the shaded squares of side x cm, as shown in the following diagram.

diagram not to scale



The remainder of the cardboard is folded up to form the tray as shown in the following diagram.

diagram not to scale



(a) Write down, **in terms of x**, the length and the width of the tray. [2]

(b) (i) State whether x can have a value of 5. Give a reason for your answer.

(ii) Write down the interval for the possible values of x. [4]

(c) Show that the volume, $V\,\text{cm}^3$, of this tray is given by

$$V = 4x^3 - 52x^2 + 160x.$$ [2]

(This question continues on the following page)

(Question 5 continued)

(d) Find $\frac{\mathrm{d}V}{\mathrm{d}x}$. [3]

(e) **Using your answer from part (d)**, find the value of x that maximizes the volume of the tray. [2]

(f) Calculate the maximum volume of the tray. [2]

(g) Sketch the graph of $V = 4x^3 - 52x^2 + 160x$, for the possible values of x found in part (b)(ii), and $0 \leq V \leq 200$. Clearly label the maximum point. [4]

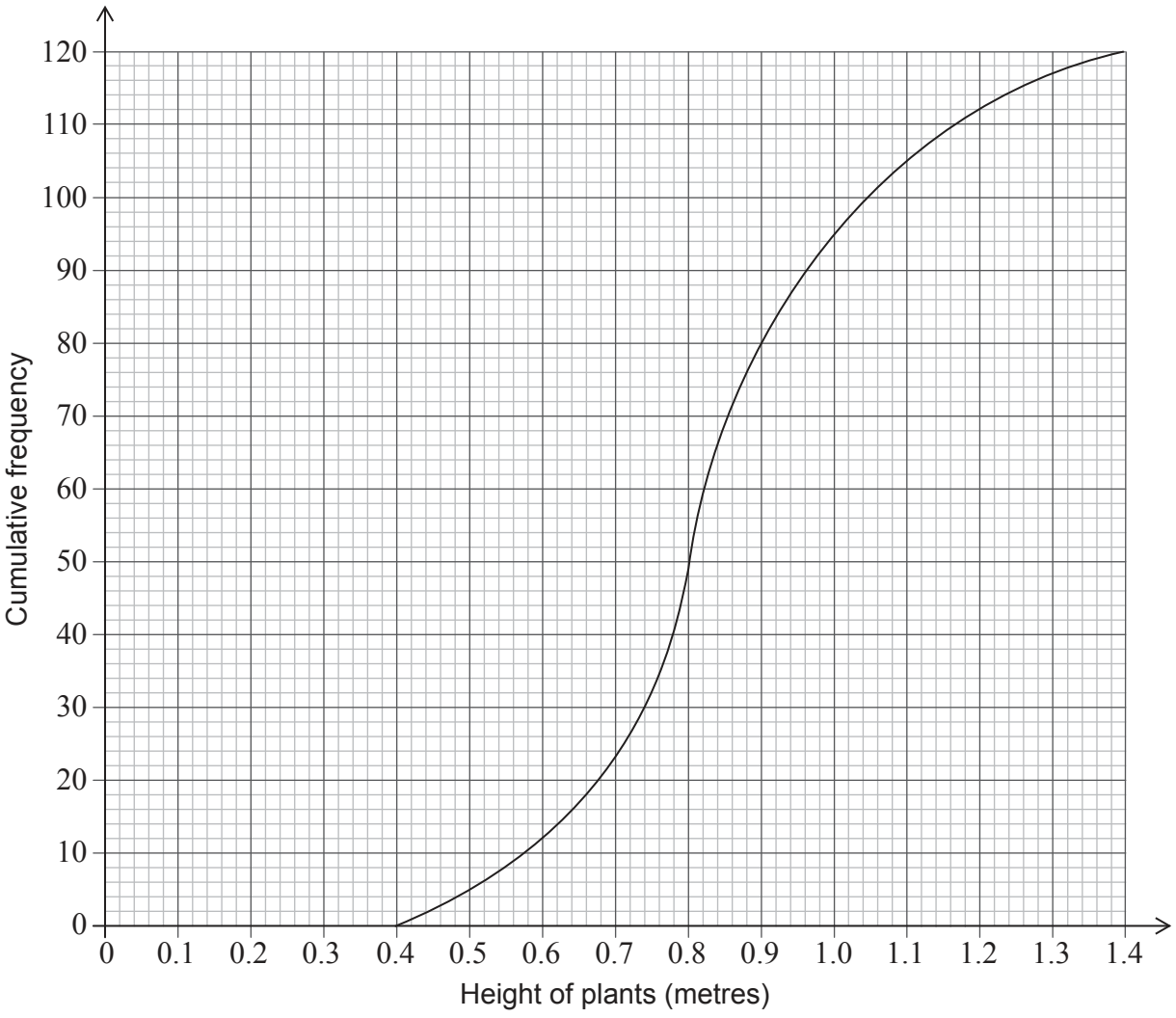
6. [Maximum mark: 15]

The table below shows the number and height (h), in metres, of plants grown for a school project.

Height, h (metres)	Frequency	Cumulative frequency
$0.40 \leq h < 0.60$	12	12
$0.60 \leq h < 0.80$	37	49
$0.80 \leq h < 1.00$	46	p
$1.00 \leq h < 1.20$	17	112
$1.20 \leq h < 1.40$	8	120

(a) Write down the value of p. [1]

This information is shown in the following cumulative frequency curve.



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(Question 6 continued)

(b) Use the graph to find the median height of the plants. [1]

At the end of the project, the school will offer some of the plants to a local charity, Greentrust, and will replant some others in the school garden.

All plants whose heights are above 1.14 metres will be replanted in the school garden.

(c) Use the graph to find the number of plants that will be replanted in the school garden. [3]

All plants whose heights are greater than the lower quartile and less than the upper quartile will be offered to Greentrust.

(d) Write down the number of plants that will be offered to Greentrust. [1]

The range of heights of the plants offered to Greentrust is $a < h < 0.96$.

(e) Write down the value of a. [1]

The shortest plant is 0.45 metres and the tallest plant is 1.35 metres.

(f) Draw a box-and-whisker diagram for this data, on graph paper, using a scale of $1\,\text{cm}$ to represent 0.1 metres. [4]

Greentrust received a total of 180 plants from local schools and decided to sell them at a market. Greentrust paid 12 euros for a market stall from which to sell the plants. At the end of the day, Greentrust made a profit of 420 euros.

(g) Calculate the selling price of one plant, in euros, if $\frac{3}{4}$ of them were sold and all plants had the same selling price. [4]